Math 216 Exam #2 Solutions

Fall 2006 - Hartlaub

#1. a. $Y_{ij} = \theta + \tau_j + \varepsilon_{ij}$ $\quad i = 1, \ldots, 6$, $j = 1, \ldots, 4$

Y_{ij} = # of cereal leaf beetles ~~Am~~ trapped for the i^{th} board of color j.

θ = overall median # of trapped cereal beetles

τ_j = the effect of the j^{th} color

ε_{ij} = random error term from a continuous distribution with median zero.

Ariela.
-2, no context
-3, just say 1-way ANOVA with $n=6$ & $K=4$

b. The medians are:

Yellow :	46.5
White :	15.5
Green :	34.5
Blue :	15.0

Yellow and green appear to be more effective than white and blue.

c. H_0: $[\tau_1 = \tau_2 = \tau_3 = \tau_4]$ no treatment effect - distribution of the trapped beetle count is the same for all colors.

vs H_1: [not all τ_j are equal] — the count is systematically higher for some colors.

Test Statistic: Kruskal-Wallis $H = 16.98$ (adjusted for ties)

P-value = .001

Since $.001 < \alpha = .01$ we have strong evidence that color affects the insect count. In other words, the difference we observed is statistically significant.

-5 no conclusion
-3, no conclusion in context.

d. $H \approx \chi^2_{3\,d.f.}$ $\Rightarrow$ the approximate P-value is between .0005 and .001 according to the χ^2 table.

$P(\chi^2_3 \geq 16.98) \approx .000713$

We would make the same conclusion with the L.S.A. Even though the sample sizes are small, the L.S.A. works well.

-6, LSA is conservative so do not reject.
2 use CV test, L

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#1 (e) Since we reject H_0 with the Kruskal-Wallis statistic, multiple comparisons should be used. $K=4$, $n_1=n_2=n_3=n_4=6$

$$\left\{\begin{array}{l}\text{decide } \tau_u \neq \tau_v \text{ if } |W^*_{uv}| \geq q_\alpha \\ \quad \text{otherwise decide } \tau_u = \tau_v\end{array}\right.$$

$\leftarrow$ Table A.17 $\left\{ q_{.05} = 3.633 \right\}$

$$W^*_{uv} = \frac{W_{uv} - \frac{6(6+6+1)}{2}}{\sqrt{\frac{6\times6(6+6+1)}{24}}} = \frac{W_{uv} - 39}{\sqrt{19.5}}$$

-5, say that MC procedures should be used, but see MTB. don't do the computations.

$W_{12} = 6\times9.5 = 57$ $\quad W^*_{12} = 4.0762$ ** $\Rightarrow \tau_1 \neq \tau_2$ Yellow $\neq$ White

$W_{13} = 6\times9.17 = 55$ $\quad W^*_{13} = 3.6233 \Rightarrow \tau_1 = \tau_3$

$W_{14} = 6\times9.5 = 57$ $\quad W^*_{14} = 4.0762$ ** $\Rightarrow \tau_1 \neq \tau_4$ Yellow $\neq$ Blue

$W_{23} = 6\times4 = 24$ $\quad W^*_{23} = -3.3968 \Rightarrow \tau_2 = \tau_3$

$W_{24} = 6\times6.83 = 41$ $\quad W^*_{24} = 0.4529 \Rightarrow \tau_2 = \tau_4$

$W_{34} = 6\times9 = 54$ $\quad W^*_{34} = 3.3968 \Rightarrow \tau_3 = \tau_4$

-3 incorrect conclusion

Summary

15.0	15.5	34.5	46.5
Blue (4)	White (2)	Green (3)	Yellow (1)

-1 incorrect lines summary

Yellow is significantly different from white + blue boards. None of the other comparisons is significant.

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#1. (f) $\theta = \left(\frac{\tau_1+\tau_2}{2}\right) - \left(\frac{\tau_3+\tau_4}{2}\right) = \frac{1}{2}\tau_1 + \frac{1}{2}\tau_2 - \frac{1}{2}\tau_3 - \frac{1}{2}\tau_4$

$n_1=n_2=n_3=n_4=6$

$N=24$

-8 suggest using Fligner-Wolfe

$Z_{12} = 31$ $\quad Z_{23} = -18.5$ $\quad Z_{34} = 18$

$Z_{13} = 13.5$ $\quad Z_{24} = 1$

$Z_{14} = 31.5$

Since $Z_{jh} = -Z_{hj}$, we know

$Z_{21} = -31$ $\quad Z_{32} = 18.5$ $\quad Z_{43} = -18$

$Z_{31} = -13.5$ $\quad Z_{42} = -1$

$Z_{41} = -31.5$

-5 do not estimate the contrast

-1 minor computational mistake

$\bar{\Delta}_h = \sum_{j=1}^{4} \frac{6 Z_{hj}}{24}$ for $h=1,2,3,4$

$\bar{\Delta}_1 = \frac{1}{4}(Z_{12}+Z_{13}+Z_{14}) = \frac{1}{4}(31+13.5+31.5) = 19$

$\bar{\Delta}_2 = \frac{1}{4}(Z_{21}+Z_{23}+Z_{24}) = \frac{1}{4}(-31+(-18.5)+1) = -12.125$

$\bar{\Delta}_3 = \frac{1}{4}(Z_{31}+Z_{32}+Z_{34}) = \frac{1}{4}(-13.5+18.5+18) = 5.75$

$\bar{\Delta}_4 = \frac{1}{4}(Z_{41}+Z_{42}+Z_{43}) = \frac{1}{4}(-31.5+(-1)+(-18)) = -12.625$

$\hat{\theta} = \sum_{j=1}^{4} a_j \bar{\Delta}_j = \frac{1}{2}(19) + \frac{1}{2}(-12.125) - \frac{1}{2}(5.75) - \frac{1}{2}(-12.625)$

$= \boxed{6.875}$

estimate will depend on a_i's chosen!

$a_1 = a_2 = 1$, $a_3 = a_4 = -1 \Rightarrow \hat{\theta} = 13.75$

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Fall 2000 - Hartlaub.

#2. (a) $Y_{ij} = \theta + \tau_j + \varepsilon_{ij}$ $\quad i = 1,2,3,4$ $\quad j = 1,2,3,4$

-2 no context
-3 just say 1-way ANOVA with n=4 and K=4.

Y_{ij} = count of CFUs for the i^{th} agar plate in season j.

θ = overall median # of CFUs

τ_j = the effect of the j^{th} season

ε_{ij} = random error term from a continuous distribution with median zero.

(b) $H_0: [\tau_W = \tau_F = \tau_{SP} = \tau_{summer}]$ - no season effect.

$H_1: [\underset{(1)}{\tau_{Winter}} \le \underset{(2)}{\tau_{Fall}} \le \underset{(3)}{\tau_{SPRING}} \le \underset{(4)}{\tau_{summer}}] \to$ with at least one <

-2 do not identify subscripts.
-3, misspecify H_1

(c) Yes, the alternative is an umbrella alternative with peak in summer (at 4).

(d) Jonckheere-Terpstra Test Statistic

$$J = \sum_{u=1}^{3}\sum_{v=2}^{v} U_{uv} = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$= 16 + 16 + 16 + 9 + 14 + 11 = 82$$

From TABLE A.13 $j_{.0041} = 76$. Thus, the p-value is less than .0041 which indicates a statistically significant effect for season

-5, use Kruskal-Wallis
-3 correct idea with incorrect labels/order.

(e) Since the Jonckheere-Terpstra statistic shows a significant effect, MC procedures should be used.

decide $\tau_u > \tau_v$ if $W^*_{uv} \ge c^*_{uv}$ ← Table A.18 (not available)

d_α ← (Table A.19) $d_{.05} = 3.295$ or $d_{.1} = 2.873$

Otherwise decide $\tau_u = \tau_v$.

-5 recognized that MC procedures should be used, but did not do the computations.

$$W^*_{uv} = [redacted] \left[\frac{W_{uv} - \frac{4(4+4+1)}{2}}{\sqrt{\frac{4\times 4\,(4+4+1)}{24}}} \right] = [redacted] \left[\frac{W_{uv} - 18}{\sqrt{6}} \right]$$

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At the $\alpha = .1$ level

#2 (e) See MTB.

$W_{12} = 6.5 \times 4 = 26$ $W^*_{12} = 3.266 *$ $\Rightarrow$ $\tau_1 < \tau_2$

$W_{13} = 6.5 \times 4 = 26$ $W^*_{13} = 3.266 *$ $\Rightarrow$ $\tau_1 < \tau_3$

$W_{14} = 6.5 \times 4 = 26$ $W^*_{14} = 3.266 *$ $\Rightarrow$ $\tau_1 < \tau_4$

$W_{23} = 4.75 \times 4 = 19$ $W^*_{23} = .4082$ $\Rightarrow$ $\tau_2 = \tau_3$

$W_{24} = 6 \times 4 = 24$ $W^*_{24} = 2.4495$ $\Rightarrow$ $\tau_2 = \tau_4$

$W_{34} = 5.25 \times 4 = 21$ $W^*_{34} = 1.2247$ $\Rightarrow$ $\tau_3 = \tau_4$

At the $\alpha = .05$ level, none of the differences are detected, so bump α up to .1.

#3. (a) $Y_{ij} = \theta + \beta_i + \tau_j + \varepsilon_{ij}$ $i = 1, \ldots, 35$ $j = 1, 2, 3, 4$

-2 no context
-3 no model

Y_{ij} = score for the i^{th} baby using condition j.
θ = overall median score
β_i = the effect of the i^{th} baby (block)
τ_j = the effect of the j^{th} condition
ε_{ij} = random error term from a continuous distribution with median 0.

b. H_0: [$\tau_1 = \tau_2 = \tau_3 = \tau_4$] distribution of score is the same for all conditions
H_a: [not all τ's equal] the score is systematically higher for some conditions

-3 use Mack-Skillings

Test stat: $S = 1.34$ d.f. $= 3$ P-value $= .719$ (adjusted for ties)

Since $.719 > .05$ we cannot reject H_0. The distribution of score is the same for all four conditions.

-2 run MC procedures.

c. Since we did not reject H_0 with the Friedman procedure, there is no need to conduct multiple comparisons. The purpose of MC procedures is to identify the significant differences

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\# 4. (a) $H_0: \tau = 0$ Kendall's $\tau = 0 \Rightarrow$ no association

$H_1: \tau > 0$ Kendall's $\tau > 0 \Rightarrow$ positive association

	Length	Weight	Concordant (# above)	Discordant (# below)	Tie
1	8.8	5.9	11	0	
2	19.2	100	10	0	
3	22.5	110	9	0	
4	23.5	120	8	0	
5	24	150	6	1	
6	25.5	145	6	0	
7	28.7	300	4	0	1
8	30.1	300	4	0	
9	39.0	685	2	1	
10	41.4	650	2	0	
11	42.5	820	1		
12	46.6	1000			
			63	2	1

-5, n incorrect, K incorrect, p incorrect.

-3 all concordant

-2 no ties correction

ties correction on p. 366 (count as 0.)

$K = 63 - 2 = 61$

$K_{.001} = 44$ from Table A.30 $K_{0.000} = 46$

$\Rightarrow$ P-value = 0.000

Since P-value = 0.000 < .01 we reject H_0 and conclude that there is a significant positive association between length & weight

(b) $\hat{\tau} = \frac{2K}{n(n-1)} = \frac{2(61)}{12(11)} = .9242 \Rightarrow$ there is a very strong positive association between length & weight.

-2 no interpretation

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#4 (c) The linear transformations from the metric system to the English system would not change the relative standing of the observations in an ordered list. Thus, the # of concordant and discordant pairs would stay the same, so the value of the correlation coefficient would stay the same. This is obviously a desirable property; we don't want the measure of association to change when the units change.

-2 generic description about relationship to mean & changing units

#5 (a) $Y_{ij} = \theta + \tau_j + \varepsilon_{ij}$ $\quad i = 1, \dots, n_j$ $\quad j = 1, \dots, 12$ $\quad (n_j = 5,\ j = 1,2,3,4,5,7,8,9,10,11,12$; $n_6 = 10)$

Y_{ij} = corticosterone level for the i^{th} mouse at time period j.

τ_j = the effect of time period j

ε_{ij} = random error term from a continuous distribution with median 0.

θ = overall median corticosterone level

-2 not context.

(b) $H_0: [\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \tau_8 = \tau_9 = \tau_{10} = \tau_{11} = \tau_{12}]$

$H_1: [\tau_1 \le \tau_2 \le \tau_3 \le \tau_4 \le \tau_5 \le \tau_6 \ge \tau_7 \ge \tau_8 \ge \tau_9 \ge \tau_{10} \ge \tau_{11} \ge \tau_{12}]$
with at least 1 strict inequality.

-1 peak at τ_5.

(c) $A^* = \dfrac{817 - 587.5}{64.983972} = 3.53164$ see attached computations

p-value $\approx 1 - .9998 = .0002$

Since $.0002 < .05$, we reject H_0 and conclude that the corticosterone level follows an umbrella pattern with known peak at $l = 6$. (time period 6).

10, used Kruskal-Wallis

-7, not significant, peak not at 6.

-5 set up correctly [two incorrect U's]

-1, error computing Var(A^*)

-.5, one U is incorrect.

4-6-90
Experiment # 3. corticosterone. (P1)

#5 Known peak at $l=6$

$H_0: \tau_1 = \cdots = \tau_{12}$

$H_1: \tau_1 \leq \cdots \leq \tau_5 \leq \tau_6 \geq \tau_7 \geq \cdots \geq \tau_{12}$

	t_1	t_2	t_3	t_4	t_5	t_6
	25	25	25	76	110	38
	25	35	80	100	110	45
	30	38	90	165	115	60
	97	80	240	165	115	60
	127	90	250	195	215	70
						75
						165
						170
						180
						180
$\bar{x}$	60.8	53.6	137	~~140.2~~ 140.2	133	104.3

	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
	50	40	25	25	25	25
	80	43	25	27	25	25
	100	54	100	27	25	25
	135	80	115	54	35	25
	155	100	125	80	90	37
$\bar{x}$	104	63.4	78	42.6	40	27.4

$n_6 = 10$

$n_1 = n_2 = n_3 = n_4 = n_5 = n_7 = n_8 = n_9 = n_{10} = n_{11} = n_{12} = 5$

$l=6$

$$A = \sum\sum_{1 \leq u < v \leq l} U_{uv} + \sum\sum_{l \leq u < v < k} U_{vu}$$

$$= 817$$

$N_1 = \sum_{i=1}^{6} n_i = 5(5) + 10 = 35$

$N_2 = \sum_{i=6}^{12} n_i = 10 + 6(5) = 40$

$N = \sum_{i=1}^{12} n_i = 11(5) + 10 = 65$

$\sum_{i=1}^{12} n_i^2 = 11(25) + 100 = 375$

$\sum_{i=1}^{12} n_i^3 = 11(125) + 1000 = 2375$

#5

C. Computation of Mann-Whitney Counts :

$U_{12} = 4.5 + 4.5 + 4 + 0 + 0 = 13$
$U_{13} = 4.5 + 4.5 + 4 + 2 + 2 = 17$
$U_{14} = 5 + 5 + 5 + 4 + 3 = 22$
$U_{15} = 5 + 5 + 5 + 5 + 1 = 21$
$U_{16} = 10 + 10 + 10 + 4 + 4 = 38$

$U_{23} = 4.5 + 4 + 4 + 3.5 + 2.5 = 18.5$
$U_{24} = 5 + 5 + 5 + 4 + 4 = 23$
$U_{25} = 5 + 5 + 5 + 5 + 5 = 25$
$U_{26} = 10 + 10 + 9.5 + 4 + 4 = 37.5$

$U_{34} = 5 + 4 + 4 + 0 + 0 = 13$
$U_{35} = 5 + 5 + 5 + 0 + 0 = 15$
$U_{36} = 10 + 4 + 4 + 0 + 0 = 18$

$U_{45} = 5 + 5 + 1 + 1 + 1 = 13$
$U_{46} = 4 + 4 + 3.5 + 3.5 + 0 = 15$

$U_{56} = 4 + 4 + 4 + 4 + 0 = 16$

Computation of Reverse Mann-Whitney Counts

$U_{12\,11} = 3.5 + 3.5 + 3.5 + 3.5 + 1 = 15$
$U_{12\,10} = 4.5 + 4.5 + 4.5 + 4.5 + 2 = 20$
$U_{12\,9} = 4 + 4 + 4 + 4 + 3 = 19$
$U_{12\,8} = 5 + 5 + 5 + 5 + 5 = 25$
$U_{12\,7} = 5 + 5 + 5 + 5 + 5 = 25$
$U_{12\,6} = 10 + 10 + 10 + 10 + 10 = 50$

$U_{11\,10} = 4.5 + 4.5 + 4.5 + 2 + 0 = 15.5$
$U_{11\,9} = 4 + 4 + 4 + 3 + 3 = 18$
$U_{11\,8} = 5 + 5 + 5 + 5 + 1 = 21$
$U_{11\,7} = 5 + 5 + 5 + 5 + 3 = 23$
$U_{11\,6} = 10 + 10 + 10 + 10 + 4 = 44$

$U_{10\,9} = 4 + 3 + 3 + 3 + 3 = 16$
$U_{10\,8} = 5 + 5 + 5 + 2.5 + 1.5 = 19$
$U_{10\,7} = 5 + 5 + 5 + 4 + 3.5 = 22.5$
$U_{10\,6} = 10 + 10 + 10 + 8 + 4 = 42$

$U_{98} = 5 + 5 + .5 + 0 + 0 = 10.5$
$U_{97} = 5 + 5 + 2.5 + 2 + 2 = 16.5$
$U_{96} = 10 + 10 + 4 + 4 + 4 = 32$

$U_{87} = 5 + 5 + 4 + 3.5 + 2.5 = 20$
$U_{86} = 9 + 9 + 8 + 4 + 4 = 34$

$U_{76} = 8 + 4 + 4 + 4 + 4 = 24$

#5 $E_0(A) = \dfrac{35^2 + 40^2 - 375 - 100}{4} = 587.5$

P3

$$Var_0(A) = \frac{1}{72}\left\{2(35^3 + 40^3) + 3(35^2 + 40^2) - 2(2{,}375) - 3(375) - 100(23) + 12(10)(35)(40) - 12(100)(65)\right\}$$

$$= 4{,}222.91667$$

$$\sqrt{Var_0(A)} = 64.983972$$

$$A^* = \frac{817 - 587.5}{64.983972} = 3.53164$$

$$\underline{\alpha} = 1 - .9998 = .0002$$

$\Rightarrow$ Reject H_0.